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Quantum-behaved Particle Swarm Optimization With Elitist Mean Best Position

Maolong Xi, Jun Sun and Wenbo Xu

Abstract — **Quantum-behaved Particle Swarm Optimization (QPSO) algorithm is a global convergence guaranteed algorithms, which outperforms traditional PSOs in search ability as well as having fewer parameters to control. In this paper, in order to depict the thinking model of people accurately that the decision-making is always influenced by the important part factors which we called elitist, so elitist mean best position is developed in QPSO to balance the global searching ability and convergence speed, and proposes a revised QPSO algorithms (EQPSO). After that, the revised QPSO algorithm is tested on several benchmark functions compared with standard QPSO and the experiment results show its superiority.**

Index Terms— **convergence speed, Elitist Mean Best Position, QPSO**

I. Introduction

Particle Swarm Optimization (PSO), motivated by the collective behaviors of bird and other social organisms, is a novel evolutionary optimization strategy introduced by J.Kennedy and R. Eberhart in 1995 [1], which relies on the exchange of information between individuals. Each particle flies in search space with a velocity, which is dynamically adjusted according to its own flying experience and its companions' flying experience. PSO's performance is comparable to traditional optimization algorithms such as simulated annealing (SA) and the genetic algorithm (GA) [2], [3]. Since its origin in 1995, many revised versions of PSO have been proposed to improve the performance of the algorithm. In 1998, Shi and Eberhart introduced inertia weight W into evolution equation to accelerate the convergence speed [4]. In 1999, Clerc employed Constriction Factor K to guarantee convergence of the algorithm and release the limitation of velocity [5]. Ozcan in 1999 and Clerc in 2002 did trajectory analysis of PSO respectively [6], [7]. In 2004, Jun Sun et al. introduce quantum theory into PSO and propose a Quantum-behaved PSO (QPSO) algorithm, which can find optimal solution in search space [8][9]. The experiment results on some widely used benchmark functions show that the QPSO works better than standard PSO and should be a promising algorithm. In recent years, many-revised QPSO algorithms with better performance are proposed [10][11][12][13].

Maolong Xi is with the Centre of Intelligent and High Performance Computing, School of Information Technology Southern Yangtze University, Wuxi, China, 214122. He is with Wuxi Institute of Technology, Wuxi, China, 214121(e-mail:wx_xml@hotmail.com)
Jun Sun is with the Centre of Intelligent and High Performance Computing, School of Information Technology Southern Yangtze University, Wuxi, China, 214122.
Wenbo Xu is with the Center of Intelligent and High Performance Computing, School of Information Technology Southern Yangtze University, Wuxi, China, 214122

In this paper, in order to depict the thinking model of people accurately, we introduce elitist mean best position in QPSO and proposed a revised Quantum-behaved Particle Swarm Optimization algorithm (EQPSO). The rest part of the paper is organized as follows. In Section 2, a brief introduction of QPSO is given. In Section 3, we introduce the revised QPSO and show how to balance the searching abilities while guaranteeing the better convergence speed of particles. Some experiments result on benchmark functions are presented in Section 4 and the results are given. Finally, the paper is concluded in Section 5.

II. Quantum-behaved Particle Swarm Optimization

Quantum-behaved particle swarm optimization is a stochastic optimization algorithm that was originally motivated by the thinking model of an individual of the social organism. In [8], Jun Sun et al considered a social organism is a system far more complex than that formulated by particle swarm optimization (PSO), and a linear evolvement equation is not sufficient to depict it at all. In practice, the evolution of man's thinking is uncertain to a great extent somewhat like a particle having quantum behavior and they introduce quantum theory into PSO and propose a Quantum-behaved PSO algorithm. The experiment results indicate that the QPSO works better than standard PSO on several benchmark functions and it is a promising algorithm in [8].

In Quantum-behaved Particle Swarm Optimization, the particle moves according to the following equations:

$$mbest = \sum_{i=1}^{M} pbest_i / M = \left(\sum_{i=1}^{M} pbest_{i1} / M, \ , \sum_{i=1}^{M} pbest_{i2} / M, \ldots \sum_{i=1}^{M} pbest_{id} / M \right) \quad (1)$$

$$p = (\varphi_1 * pbest + \varphi_2 * gbest) / (\varphi_1 + \varphi_2) \quad (2)$$

$$x(t+1) = p \pm \beta * \left| mbest - x(t) \right| * \ln(1/u) \quad (3)$$

Where

pbest The local best position among the particle (best information of a individual)

gbest The global best position among the particle (best information of all particles in the population)

mbest The mean best position among the particle, which is calculated by equation (1).

M The population size

p A stochastic point between *pbest* and *gbest*

φ_1 φ_2 A random number distributed uniformly on [0,1]

u Another random number distributed uniformly on [0,1]

β The parameter of QPSO, called Contraction-Expansion Coefficient.

$\pm$ is decided by a random number between 0 and 1 in every iteration, when the number is bigger than 0.5, - is used otherwise + is used

III. LINEAR WEIGHT QUANTUM-BEHAVED PARTICLE SWARM OPTIMIZATION

We can find that a mainstream thought point is employed to evaluate the creativity of a particle and the point and Mean Best Position *(mbest*) is defined as the center-of –gravity *gbest* position of the particle swarm which is formulated as equation (1). That is to say the coefficient of every particle is 1 and they have the same importance to *mbest*.

In practice, we know that the importance of information in our brain is not equal to each other and relate to the problem and other factors. The decision in society is make by part people called elitist. Therefore, the center-of –brain gravity *gbest* position of the particle swarm depict as equation (1), which has fixed weight, is not accord to the fact and should assign part elitist particles to generate *mbest*.

In this paper, we consider the elitist particles are the top best particles, which have bigger numerical value and formulate as equation (4).

$$mbest = \sum_{i=1}^{N} pbest_i / N = (\sum_{i=1}^{N} pbest_{i1} / N, \sum_{i=1}^{N} pbest_{i2} / N, \ldots$$
.
$$\sum_{i=1}^{N} pbest_{id} / N) \qquad (4)$$

Where N is the number of the elitist particles, which equal to two of three M.

The revised QPSO algorithm is described as follows.

```
Initialize population: random xi
do
find out mbest using equation (4)
for i=1 to population size M
If f (xi)<f(Pi) then pi=xi
pg=min (Pi)
for d=1 to dimension D
fi1=rand (0,1), fi2=rand (0,1)
P=(fi1*pid+fi2*pgd)/(fi1+fi2)
L=beta*abs(mbestd-xid)
u=rand (0,1)
if rand (0,1)>0.5
xid=P-L*ln(1/u)
else
xid=P+L*ln(1/u)
Until termination criterion is met
```

IV. EXPERIMENT RESULTS

To test the performance of revised QPSO (EQPSO), five benchmark functions listed in table 1 are used here for comparison with QPSO algorithm in [8].

These functions are all minimization problems with minimum value zeros. In all experiments, the initial range of the population also listed in Table 1 is asymmetry as used in [8~9]

TABLE 1. BENCHMARK FUNCTIONS

Functions	Formulations	Initialization	Max Range
Sphere function ***f*1**	$f_1(x) = \sum_{i=1}^{n} x_i^2$	(50,100)	100
Rosenbrock function ***f*2**	$f_2(x) = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	(15,30)	100
Rastrigrin function ***f*3**	$f_3(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	(2.56,5.12)	10
Griewank function ***f*4**	$f_4(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	(300,600)	600
De Jong's function ***f*5**	$f_5(x) = \sum_{i=1}^{n} ix_i^4$	(30,100)	100

The fitness value is set as function value and the neighborhood of a particle is the whole population. We had 100 trial runs for every instance and recorded mean best fitness and standard variance. In order to investigate the scalability of the algorithm, different population sizes *M* are used for each function with different dimensions. The population sizes are 20, 40 and 80 and the maximum generation is set as 1000, 1500 and 2000 corresponding to the dimensions 10, 20 and 30 for five functions, respectively. We make two groups of experiments to test the QPSO and the weight QPSO (EQPSO). In the first set of experiments, the QPSO is tested, and the coefficient β decreases from 1.0 to 0.5 linearly when the algorithm is running as in [9]. The second set is to test the performance of EQPSO with the N

equal to two of three M, which are 14,27,54 respectively. The mean best fitness values for 100 runs of each function in Table 2 to Table 5.

TABLE 2. THE MEAN FITNESS VALUE FOR SPHERE FUNCTION AND ROSENBROCK FUNCTION

M	Dim.	Ger.	SPHERE FUNCTION				ROSENBROCK FUNCTION			
			QPSO		EQPSO		QPSO		EQPSO	
			Mean Best	St.Var.	Mean Best	St.Var.	Mean Best	St.Var..	Mean Best	St.Var.
20	10	1000	7.6062e-041	7.683e-043	1.185e-046	1.197e-048	40.455	0.35225	71.091	2.1914
	20	1500	9.2651e-016	9.3587e-018	1.912e-024	1.2961e-026	79.43	0.67274	137.28	1.1962
	30	2000	2.0825e-014	2.1029e-016	1.6168e-014	1.6076e-016	101.54	0.87542	112.27	0.91177
40	10	1000	9.1026e-075	9.1945e-077	9.8369e-082	9.9363e-084	18.114	0.16288	19.549	0.56975
	20	1500	4.2773e-042	4.3203e-044	6.4697e-046	6.5315e-048	56.688	0.45684	61.447	0.47855
	30	2000	7.4825e-030	7.5297e-032	5.1621e-031	5.2125e-033	79.0491	0.5386	102.8798	0.7855
80	10	1000	1.951e-101	1.9683e-103	1.0743e-118	1.0798e-120	6.8198	0.063445	9.5343	0.24817
	20	1500	2.4053e-068	2.4295e-070	1.8063e-074	1.822e-076	35.675	0.22603	35.084	0.1921
	30	2000	2.4213e-050	2.4383e-052	2.2549e-055	1.2096e-057	48.66	0.3418	62.686	0.70608

TABLE 3. THE MEAN FITNESS VALUE FOR RASTRIGRIN FUNCTION AND GRIEWANK FUNCTION

M	Dim.	Ger.	RASTRIGRIN FUNCTION				GRIEWANK FUNCTION			
			QPSO		EQPSO		QPSO		EQPSO	
			Mean Best	St.Var.	Mean Best	St.Var.	Mean Best	St.Var.	Mean Best	St.Var.
20	10	1000	4.7681	0.038112	4.8735	0.0090257	0.066486	0.00067093	0.067567	0.00030049
	20	1500	15.504	0.036004	16.082	0.10891	0.018209	5.7156e-005	0.024723	0.00013791
	30	2000	29.808	0.053429	31.96	0.12181	0.010228	0.00010331	0.012481	5.1359e-005
40	10	1000	3.3947	0.033841	2.9351	0.019598	0.057602	2.1371e-005	0.042091	0.00035045
	20	1500	10.018	0.00068714	11.263	0.077184	0.017	0.00024914	0.022871	0.00015632
	30	2000	23.6894	0.0622	22.6457	0.0780	0.0070	1.2868e-004	0.0112	3.6293e-005
80	10	1000	2.1689	0.008397	1.771	0.012261	0.040678	0.00029079	0.038305	0.00025838
	20	1500	8.2122	0.0025503	8.358	0.0040238	0.014783	2.4869e-005	0.015645	0.00023854
	30	2000	15.393	0.015367	17.203	0.057386	0.0089754	9.0661e-005	0.010201	0.00010304

TABLE 4. THE MEAN FITNESS VALUE FOR DE JONG'S FUNCTION

M	Dim.	Ger.	QPSO		EQPSO	
			Mean Best	St.Var.	Mean Best	St.Var.
20	10	1000	5.5543e-067	5.6104e-069	1.2504e-069	1.263e-071
	20	1500	3.5892e-031	3.6254e-033	1.9535e-031	1.9732e-033
	30	2000	1.6973e-017	1.7141e-019	4.2557e-017	4.2197e-019
40	10	1000	5.9703e-108	6.0306e-110	6.4859e-126	6.5515e-128
	20	1500	4.0228e-060	4.0634e-062	3.5504e-061	3.5862e-063
	30	2000	2.1344e-038	2.1559e-040	3.6865e-039	3.7237e-041
80	10	1000	7.0972e-163	7.7776e-164	7.5265e-186	0
	20	1500	8.1705e-093	8.253e-095	1.834e-100	1.8526e-102
	30	2000	3.5264e-063	3.5615e-065	1.4275e-71	3.2454e-73

Fig1 to Fig5 give the comparison of convergence processes of EQPSO and QPSO in the above five benchmark functions averaged on 100 trial runs, when the population size is 40 and the maximum generation is 2000 according to the dimension 30 for five benchmarks. The coefficient β also decreases from 1.0 to 0.5 linearly.

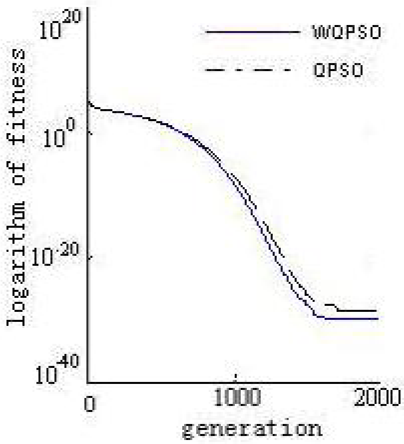

Fig. 1 Comparison of convergence process with two algorithm in Sphere function

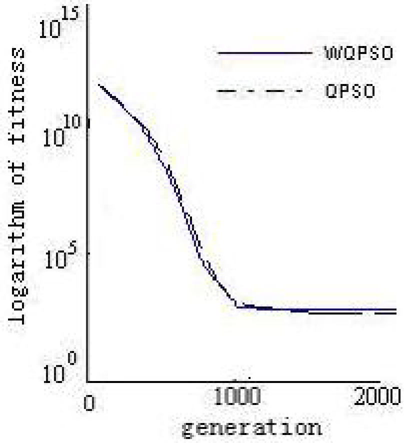

Fig. 2 Comparison of convergence process with two algorithm in Rosenbrock function

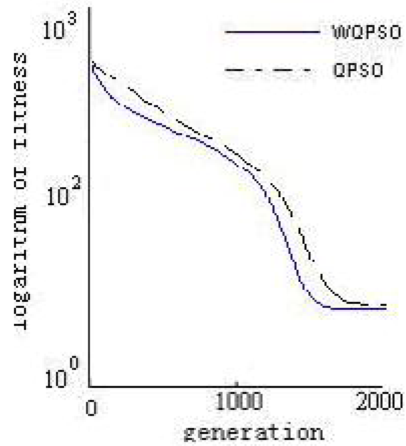

Fig. 3. Comparison of convergence process with two algorithm in Rastrigrin function

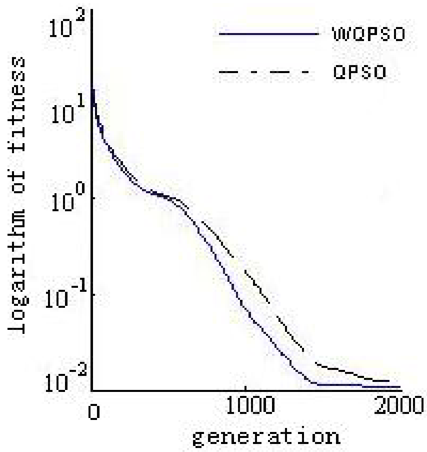

Fig. 4. Comparison of convergence process with Two algorithm in Griewank function

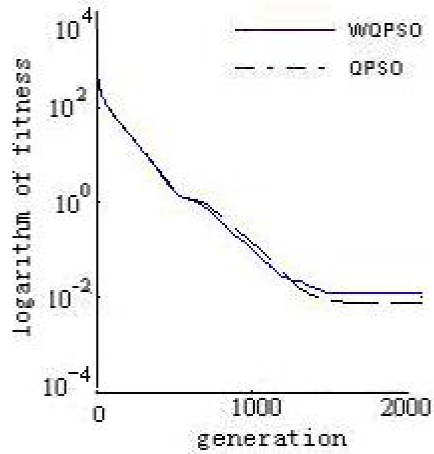

Fig. 5. Comparison of convergence process with two algorithm in De Jong's function

In table 2 and table 4, the experiment results on Sphere function and De Jong's function of EQPSO could hit the optimal solution with high precision and faster convergence speed, the smaller standard variance numerical value show that EQPSO has better stability with faster convergence. In table 3, the experiment results on Rastrigrin function and Greiwank function are approach two each other with two algorithms, and the advantage of EQPSO is not remarkable. The experiment results in table 2 on Rosenbrock test function tell us that QPSO has better performance than EQPSO.

Fig1 to Fig5 give the intuitionistic results, the convergence speed of EQPSO is faster than QPSO on five-test function in initial stage of experiment in every figure, but EQPSO has no superior to QPSO on Rosenbrock at last.

From the results above, we can conclude that the calculation method of elitist mean best position can make the convergence speed of QPSO faster which may lead to good performance of the algorithm on convex function such as Sphere and De Jong's. On complex multi-modal function optimization problems such as Rastrigrin and Griewank function, the tradeoff between global search ability and local search ability is vital to the performance of the algorithm. The slow convergence speed corresponds to good global search ability, while fast speed results in good local search ability.

V. CONCLUSIONS

In this paper, we have described the quantum-behaved particle swarm optimization and the mean best position of the population. Based on analysis the thinking model of social organism with high-level intelligence, we introduced elitist to mean best position, which can attract neighboring particles and proposed revised QPSO. This method is more approximate to the learning process of people and can make the population evolve rapidly with good global searching ability. In our future work, we will be devoted to find out a more efficient methods, and thus to enhance the performance of QPSO fatherly.

REFERENCES

[1] J.Kennedy and R.Eberhart. "Particle Swarm Optimization". *Proceedings of IEEE international Conference On Neural Network*, 1995,pp 1942~1948,

[2] Angeline, P.J.: Evolutionary Optimization Versus Particle Swarm Optimization: Philosophyand Performance Differences. *Evolutionary Programming VIII. Lecture Notes in Computer Science 1477*. Springer. 1998, pp 601 – 610

[3] Eberhart, R.C., Shi, Y.: Comparison between Genetic Algorithm and Particle Swarm Optimization. *Evolutionary Programming VII. Lecture Notes in Computer Science 1447*.Springer1998, pp 611 – 616

[4] Shi, Y., Eberhart, R.C.: A Modified Particle Swarm Optimizer. *Proceedings of the IEEE International Conference on Evolutionary Computation. Washington. DC. Piscataway. NJ: IEEE Press*. 1998, pp 69 – 73

[5] Clerc, M.: The Swarm and Queen: Towards a Deterministic and Adaptive Particle Swarm Optimization. *Proc. IEEE Congress on Evolutionary Computation*. *1999*, pp 1591 – 1597

[6] Ozcan, E., Mohan, C.K.: Particle Swarm Optimization: Surfing the Waves. *Proc. of Congress on Evolutionary Computation. 1999*, pp 1939 – 1944

[7] Clerc, M., Kennedy, J.: The Particle Swarm: Explosion, Stability and Convergence in a Multi-Dimensional Complex Space. *IEEE Transaction on Evolutionary Computation. 2002*, pp 58 – 73

[8] J. Sun, B. Feng and W. Xu, "Particle Swarm Optimization with Particles Having Quantum Behavior," *IEEE Proc. of Congress on Evolutionary Computation*, 2004. pp 325-331

[9] J.Sun, W.Xu, B.Feng, "Adaptive Parameter Control for Quantum-behaved Particle Swarm Optimization on Individual Level" *IEEE International Conference on Systems, Man and Cybernetics, 2005* pp 3049-3054

[10] Jun Sun, Wenbo Xu, Bin Ye: Quantum-Behaved Particle Swarm Optimization Clustering Algorithm. *ADMA 2006*, pp 340-347

[11] Jing Liu, Wenbo Xu, Jun Sun: Quantum-Behaved Particle Swarm Optimization with Mutation Operator. *ICTAI 2005*,v 237-240

[12] Jing Liu, Jun Sun, Wenbo Xu: Improving Quantum-Behaved Particle Swarm Optimization by Simulated Annealing. *ICIC (3) 2006*:, pp 130-136

[13] Jun Sun, Wenbo Xu, and Wei Fang: Enhancing Global Search Ability of Quantum-Behaved Particle Swarm Optimization by Maintaining Diversity of the Swarm. *RSCTC 2006*, pp 736-745